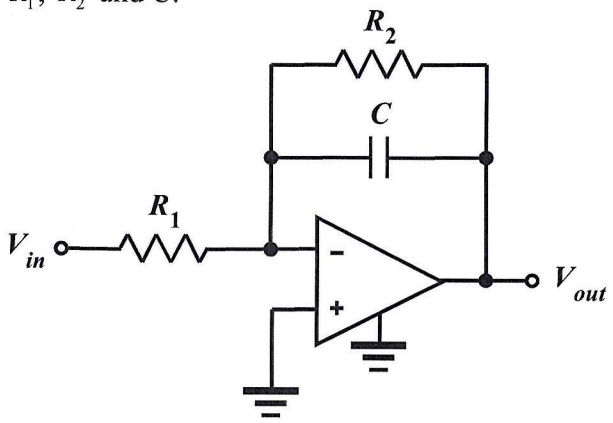
EE 3340

Homework Problem #024

a. For the active filter circuit shown below, determine an expression for the voltage transfer function in terms of R_1, R_2 and C.



$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} + \frac{V_{out}}{\left(\frac{1}{j\omega C}\right)} = 0 \Rightarrow \frac{1}{R_1} V_{in} = -\left(\frac{1}{R_2} + j\omega C\right) V_{out}$$

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{\frac{1}{R_1}}{\frac{1}{R_2} + j\omega C}$$

b. Characterize the circuit as LP or HP, and justify your decision by determining values for the low-frequency gain, the high-frequency gain, and the cutoff frequency.

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\frac{1}{R_1}}{\sqrt{\left(\frac{1}{R_2}\right)^2 + (\omega C)^2}}$$

$$\lim_{\omega \to 0} \left|\frac{V_{out}}{V_{in}}\right| = \frac{1/R_1}{1/R_2} = \frac{R_2}{R_1} \leftarrow \text{low-frequency gain}$$

$$\lim_{\omega \to \infty} \left|\frac{V_{out}}{V_{in}}\right| = 0 \leftarrow \text{high-frequency gain}$$

∴ This is a Low Pass (LP) filter.

At the cutoff frequency:

$$\frac{\frac{1}{R_1}}{\sqrt{\left(\frac{1}{R_2}\right)^2 + (\omega_c C)^2}} = \frac{1}{\sqrt{2}} \frac{R_2}{R_1} \Rightarrow \omega_c C = \frac{1}{R_2} \Rightarrow \omega_c = \frac{1}{R_2 C}$$